Persuading Voters

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Motivation

- Information is the cornerstone of democracy.
- Often it is provided by a third party.
- The third party (politician) can increase the probability of approval **by strategically designing information**.

A Hook

		states		
		a	b	c
voters	A	**+1.1**	-1	-1
	B	-1	**+1.1**	-1
	C	-1	-1	**+1.1**

Table 1: Payoffs from Approving the Proposal.

Focus

1. How does the politician **strategically design the policy experiment**?
2. How do the voting rules shape **the equilibrium payoff of voters**?
3. How do the voting rules affect **the information provision (with commitment) and the voters' welfare**?

Applications

- Voting for Public Goods
- Promotion
- Democracy Politics
- Corporate Governance

Main Findings

1. Under **a simple-majority rule**, the politician' s influence always makes a majority of voters **weakly worse off**.
2. This negative influence can happen even when voters' preferences are very **aligned**.
3. Voters face a trade-off between **control and information**.
4. When their preferences are aligned, each voter has **single-peaked preferences** over k-voting rules and even prefers **unanimity** over any other k-voting rule

Related Literature

1. **Information Design: Multiple Receivers in A Game**
   - lonso and C‘amara (2016), Michaeli (2014), Taneva (2014) and Wang (2013)
2. **Institution rule endogenously affects the information**
   - Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996) and Jackson and Tan (2013)

The Model: Voters

- $n \geq 1$: voters
- $X = \{x_1, x_0\}$: two alternatives
- Θ: finite state space with $\theta_i < \theta_{i+1}$ for all $i \in \{1, 2, ..., T\}$ and $T \geq 2$
- $p = (p_\theta)_\theta \in \Theta$: common prior belief
- $u_i(x, \theta), u_i : X \times \Theta \to R$: voters payoff function
- $\delta = (\delta^i_\theta)_{\theta \in \Theta} = (u_i(x_1, \theta) - u_i(x_2, \theta))_{\theta \in \Theta}$:the voters type

The Model: Politician and Voting Rules

1. Politician
   - Payoff function: $V(x_1, \theta) = 1$ and $V(x_0, \theta) = 0$ for all θ
   - Policy experiment;
     1. commits to public experiment $\pi = \pi(\cdot|\theta)$
     2. finite realization space S: $\pi(\cdot|\theta) \in \Delta(S)$
        (Let $q = q(s|\pi.p)$ be the updated posterior belief of voters)
2. Voting Rules
   - Proposal x_1 is selected iff it receives at least k votes, where $k \in \{1, 2, ..., n\}$ is the established electoral rule.

The Model: Equlibrium Selection

- i's expected net payoff from implementing the proposal: $\langle q, \theta \rangle = \sum_{\theta \in \Theta} \delta_\theta q_\theta \geq 0$
- optimal voting strategy: $a(q, \theta) = 1$ if $\langle q, \theta \rangle \geq 0$ and $a(q, \theta) = 0$ if $\langle q, \theta \rangle < 0$
  where $a : \Delta(\Theta)$ x $R^n \rightarrow \{0, 1\}$
- Politician's problem: maximize $E_\pi[v(q)] \Leftrightarrow$ maximize $Pr(Approval)$

- δ^i and δ^j rank states in the same order:
  for every pair $\theta, \theta^{`} \in \Theta$, we have $\delta^i_\theta > \delta^i_{\theta^{`}} \Leftrightarrow \delta^j_\theta > \delta^j_{\theta^{`}}$
- δ^i and δ^j agree under full information:
  for every $\theta \in \Theta$, we have $\delta^i_\theta \geq 0 \Leftrightarrow \delta^j_\theta \geq 0$

- policy implementation brings payoff δ_θ^i to voter i under state θ
- the win set: $W_k = \{q \in \Delta(\Theta) | \sum_{i=1}^n a(q, \theta_i) \geq k\}$
- the set of approval states: $D(\theta) = \{\theta \in \Theta | \delta_\theta \geq 0\}$
- the set of approval beliefs:$A(q) = \{q \in \Delta(\Theta) | \langle q, \theta \rangle \geq 0\}$
- the set of strong rejection beliefs:$R(q) = \{q \in \Delta(\Theta) | \theta \in D \Rightarrow q(\theta) = 0\}$
- all coalitions containg at least $n - k + 1$ voters: B
- the set of strong rejection beliefs (with a coalition):$R_k = \cup_{b \in B} \cap_{\delta \in b} R(\delta)$

- If $p \in A(p)$, no need to run an experiment
- **Proposition**:
  So suppose that $p \notin A(p)$ and $W \neq \phi$. An optimal π involves $\{s^+, s^-\}$, where s^+ induces posterior $q^+ \in A(q)$ while s^- induces posterior $q^- \in R(\delta)$. q^+ and q^- satisfies:
  1. q^+, q^- max $\frac{||q^- - p||}{||q^+ - p||}$
  2. q^+, q^-, p are collinear(Bayes Plausibility)
- Then the equilibrium approval probability:
  Pr(Approval)$=\frac{||q^+ - p||}{||q^- - p|| + ||q^+ - p||}$

- **Intuition** (Figure $|\Theta| = 3$)
  1. π max approval probability
     $\Leftrightarrow \pi$ max q^+
     $\Leftrightarrow q^+$ is more "closer" to p while q^- is more "further" away from p
  2. Collinearity: Bayes Plausibility implies that p is the convex combination of q^+ and q^-

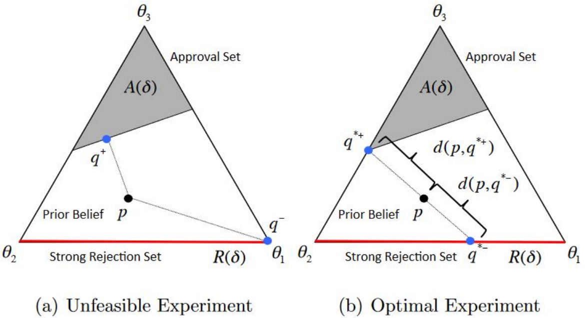
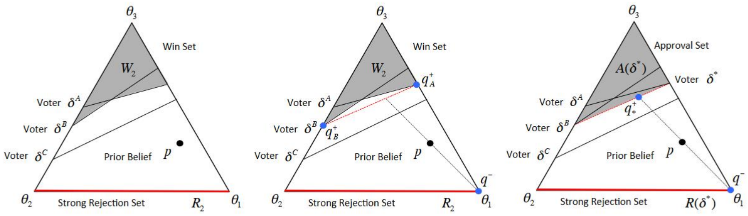


Figure 1: Simplex Representing the Beliefs of Dictator δ, with $\delta_{\theta_3} > 0 > \delta_{\theta_2} > \delta_{\theta_1}$

- **Proposition**:
  There exists a θ^* such that, for every optimal experiment:
  1. the voter approvals for any δ_θ if $\delta_\theta > \delta_{\theta^*}$
  2. the voter rejects for any δ_θ if $\delta_\theta > \delta_{\theta^*}$
  3. Indifference: the dictator is indifferent to approval and rejection
- **Intuition**: the politician bundles the rejection states with the smallest incremental loss, i.e the smallest $|\delta_\theta|$

- if $p \in co(W_k)$, then it is easy to run an experiment to successfully persuade the voters
- Suppose $p \notin co(W_k)$ and $W_k \neq \phi$.
- **Proposition**:
  An optimal π^* involves running π_1 followed by π_2 that induce the following $\tau_1, \tau_2 \in \Delta(\Delta(\Theta))$:
  1. $supp(\tau_1) = \{q^-, q^+\}$, s.t. $\tau_1 q^+ + (1 - \tau_1) q^- = p$
     - $q^+ \in co(W_k)$
     - q^- s.t. at least k voters believe that $Pr[\delta^i_\theta < 0] = 1$
     - q^+, q^- max $\frac{||q^- - p||}{||q^+ - p||}$
  2. $supp(\tau_2) \in W_k$ and $E_{\tau_2}[q] = q^+$
- **Intuition**: think of $co(W_k)$ as a single receiver and τ_2 is for forming the coalition

k-voting rule: Optimal Experiment



(a) Win Set (b) Optimal Experiment (c) Weak Representative Voter δ^*

Figure 2: Optimal Experiment for Example 2

Welfare Analysis

- let $u_k^i(p)$ be i's payoff without any π
- let $u_k^i(\pi^*)$ be i's payoff with π^*
- **Proposition**:
  1. If $k = n$, then $u_k^i(\pi^*) \geq u_k^i(p)$
  2. If $k < n$, then $u_k^i(\pi^*) \geq u_k^i(p)$ for at most $k - 1$ voters, while $u_k^i(\pi^*) \leq u_k^i(p)$ for at least $n - k + 1$ voters.
     And these at least $n - k + 1$ voters are strictly worse off if no optimal π^* satisfies $Supp|\pi^*| = 2$
- **Intuition**:
  1. Choose a less informative experiment to strictly increase the probability of approval.
  2. No optimal experiment with only two realizations implies the politician must be targeting different winning coalitions.
- Corollary: If $p \notin W_{\frac{n+1}{2}}$, then a majority of voters prefer unamanity over simple majority.

- **Prposition**:
  Suppose all voters rank in the same order, then they have single peaked preference over $k(\theta)$:
  The voter's expected utility is non-decreasing in k, for $k(\theta) > k$, while non-increasing for $k(\theta) < k$
- **Intuition**: Considering the cutoff state

The sufficient Conditions for the "Monotone" Preference

- **Proposition**:
  - Suppose all voters:
    1. rank states in the same order
    2. agree under full information
  - Then, every voter weakly prefers a $k+1$-voting rule to a k-voting rule, for $k \in \{1, 2, ..., n\}$.
  - Consequently, every voter weakly prefers unanimity over any other k-voting rule.
- **Intuition**:
  1. θ_k^* is the same cut-off state for all voters
  2. All voters view the weak representative voter as too easy to persuade and, thus, prefer a higher k rule.

Extensions

- Controller knows the State
- Controller's Payoff Depends on the State
- Preference Shocks
- Heterogenous Prior Beliefs
- Optimal Endorsement: Another Interpretation